Multivariable Calculus

Quiz 9 **SOLUTIONS**

1) Compute the double integral

$$\iint_{\mathcal{R}} \frac{1+x^2}{1+y^2}\, dA$$

over the rectangle $\mathcal{R} = [0,1] \times [0,1]$.

Solution:

$$\begin{aligned}
\iint_{\mathcal{R}} \frac{1+x^2}{1+y^2}\, dA &= \int_0^1 \left[\int_0^1 \frac{1+x^2}{1+y^2}\, dx \right] dy \\
&= \left[\int_0^1 (1+x^2)\, dx \right] \left[\int_0^1 \frac{1}{1+y^2}\, dy \right] \\
&= \left[x + \frac{x^3}{3} \right]_0^1 \left[\arctan(y) \right]_0^1 \\
&= \frac{4}{3} \cdot \frac{\pi}{4} = \frac{\pi}{3}
\end{aligned}$$

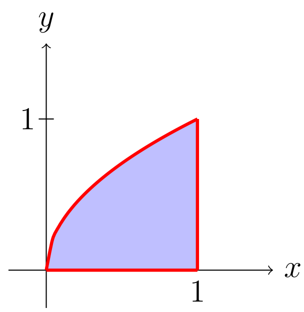
TURN OVER

2) Compute the double integral

$$\iint_{\mathcal{D}} \frac{2y}{x^2+1}\, dA$$

where $\mathcal{D}$ is the region enclosed by $y = 0, x = 1$, and $y = \sqrt{x}$ in the first quadrant.

Solution: The region we need to integrate over is shown below.



So, we can take the limits to be $0 \leq x \leq 1$ and $0 \leq y \leq \sqrt{x}$.

$$\begin{aligned}
\iint_{\mathcal{D}} \frac{2y}{x^2+1}\, dA &= \int_0^1 \left[\int_0^{\sqrt{x}} \frac{2y}{x^2+1}\, dy \right] dx \\
&= \int_0^1 \left. \frac{y^2}{x^2+1} \right|_0^{\sqrt{x}} dx \\
&= \int_0^1 \frac{x}{x^2+1}\, dx \\
&= \left. \frac{1}{2} \ln(x^2+1) \right|_0^1 \\
&= \frac{\ln(2)}{2}
\end{aligned}$$